Introduction to the Optical Indicatrix

Modified from the lecture notes of Prof. Stephen A. Nelson at Tulane University.

What is the Optical Indicatrix?

The concept of the optical indicatrix is important as a visual means of looking at the way refractive index varies with direction in a substance.

The optical indicatrix is simply a three-dimensional object constructed by drawing vectors of length proportional to the refractive index for light vibrating parallel to the vector direction from a central point. The ends of all of the vectors are then connected to form the indicatrix.

The Isotropic Indicatrix

Isotropic substances are those wherein the velocity of light or the refractive index does not vary with direction in the substance. Substances such as gases, liquids, glasses, and isometric minerals are isotropic.

For isotropic minerals the indicatrix is a sphere, as the refractive index does not vary with direction (**fig. 1**).

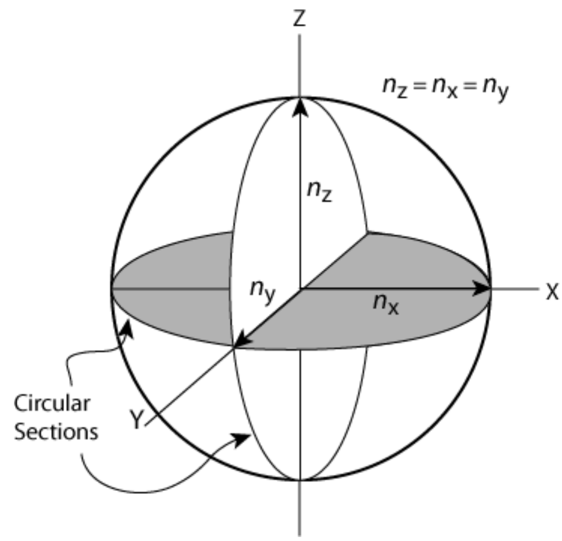

Figure 1. The isotropic indicatrix. The value of n is equal in all directions, creating an optical "sphere".

Introduction to Uniaxial Minerals

Uniaxial minerals are a class of anisotropic minerals that include all *tetragonal* and *hexagonal/rhombohedral* minerals. They are called uniaxial because they have a single **optic axis**. Light traveling along the direction of this single optic axis exhibits the same properties as isotropic materials in the sense that *the polarization direction of the light is not changed by passage through the crystal*. Similarly, if the optic axis is oriented perpendicular to the microscope stage with the analyzer inserted, the grain will remain extinct throughout a 360° rotation of the stage.

The single optic axis is coincident with the *c-crystallographic axis* in uniaxial minerals. Thus, light traveling parallel to the c-axis will behave as if it were traveling in an isotropic substance because, looking down the c-axis of tetragonal or hexagonal minerals one sees only equal length a-axes, just like in isometric minerals (**fig. 2**).

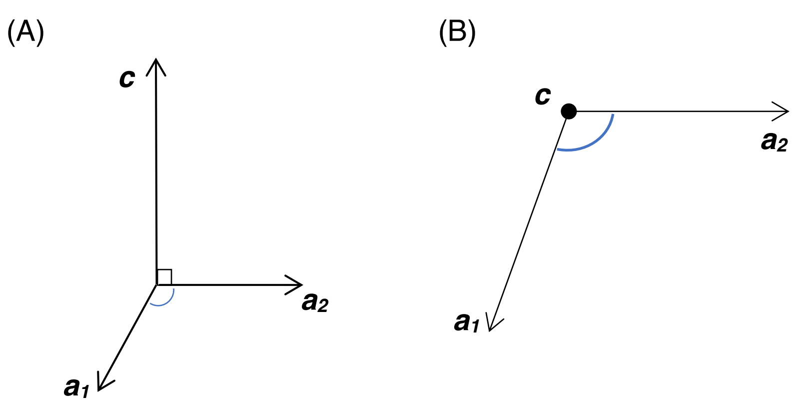

Figure 2. The hexagonal crystal system (pictured) and tetragonal crystal system are **uniaxial**. When viewed down the ***c*** axis (B), the a_1 and a_2 axes, which are optically equal, make the mineral *appear* isotropic. When viewed from other angles (A), the mineral displays interference colors due to birefringence.

Like all anisotropic substances, the refractive indices of uniaxial crystals varies between two extreme values. For uniaxial minerals these two extreme values of refractive index are defined as ω (or n_ω) and ε (or n_ε). Values between ω and e are referred to as ε'.

Uniaxial minerals can be further divided into two classes. If ω > ε the mineral is said have a negative optic sign or is uniaxial negative. In the opposite case, where ε > ω the mineral is said to have a positive optic sign or is uniaxial positive. The absolute birefringence of a uniaxial minerals is defined as l ω - ε l (the absolute value of the difference between the extreme refractive indices).

Uniaxial Indicatrix

Just like in isotropic minerals, we can construct an indicatrix for uniaxial minerals. The **uniaxial indicatrix** is constructed by first orienting a crystal with its c-axis vertical. Since the c-axis is also the optic axis in uniaxial crystals, light traveling along the c-axis will vibrate perpendicular to the c-axis and parallel to the ω refractive index direction. Light vibrating perpendicular to the c-axis is associated with the ω-ray as discussed above. Thus, if vectors are drawn with lengths proportional to the refractive index for light vibrating in that direction, such vectors would define a circle with radius ω. This circle is referred to as the circular section of the uniaxial indicatrix.

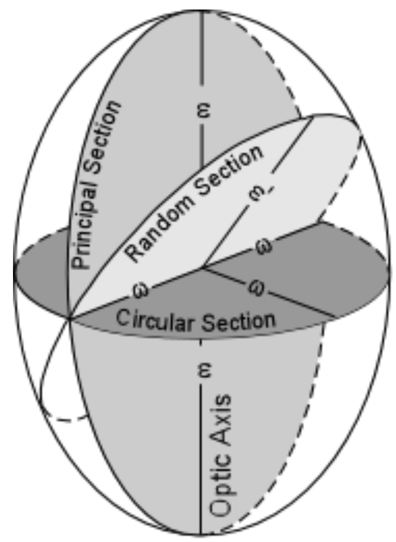

Figure 3. The uniaxial indicatrix. The axes of the ellipsoid are equivalent to the crystallographic axes. ***c*** ≡ ε and ***a₁*** ≡ ***a₂*** ≡ ω.

Light propagating along directions perpendicular to the c-axis or **optic axis** is broken into two rays that vibrate perpendicular to each other. One of these rays, the ε-ray vibrates parallel to the c-axis or optic axis and thus vibrates parallel to the ε refractive index. Thus, a vector with length proportional to the ε refractive index will be larger than or smaller than the vectors drawn perpendicular to the optic axis, and will define one axis of an ellipse. Such an ellipse with the ε direction as one of its axes and the ω direction as its other axis is called the **Principal Section** of the uniaxial indicatrix.

Light vibrating parallel to any direction associated with a refractive index intermediate between ε and ω will have vector lengths intermediate between those of ε and ω and are referred to as ε' directions. Light propagating along one of the ε' directions is broken into two rays, one vibrates parallel to an ε' direction and the other vibrates parallel to the ω direction. An ellipse that has an ε' direction and a ω direction as its axes is referred to as a **random section** of the indicatrix.

Optic Sign

Uniaxial minerals can be divided into 2 classes based on the optic sign of the mineral (**fig. 4**).

- If ω > ε, the optic sign is **negative** and the uniaxial indicatrix would take the form of an oblate spheroid. *Hint: this indicatrix is elongated in the direction of the stroke of a minus sign.*

- If ω < ε, the optic sign is **positive** and the uniaxial indicatrix would take the form of a prolate spheroid. *Hint: this indicatrix is elongated in the direction of the vertical stroke of a plus sign.*

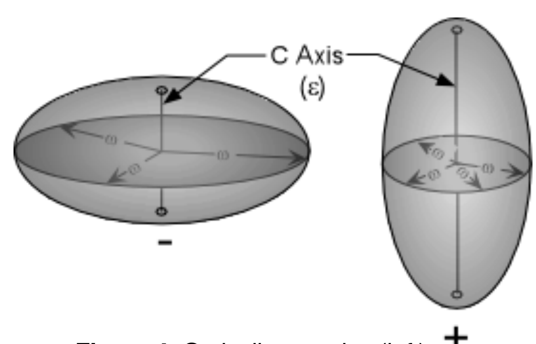

Figure 4. Optically negative (left) and optically positive (right) uniaxial indicatrices.

Application of the Uniaxial Indicatrix

This is shown in **fig. 5** for an imaginary tetragonal crystal. In this case the optic sign of the mineral is positive, and the uniaxial indicatrix is shown at the center of the crystal.

If the crystal is mounted on the microscope stage such that the c-axis or optic axis is perpendicular to the stage, we can move the indicatrix up to the top face of the crystal (face a) and see that such light will be vibrating in the ω direction even if we rotate the stage. Thus we will see the circular section of the indicatrix.

If the crystal is mounted on the stage such that the c-axis is parallel to the stage, we can move the indicatrix to one of the side faces of the crystal (such as face c) and see that light will be broken up into two rays, one vibrating parallel to the ε direction and one vibrating parallel to the ω direction. Thus we will see one of the principal planes of the indicatrix.

If the crystal is mounted on the stage such that the c-axis or optic axis is neither parallel to or perpendicular to the stage, we can move the indicatrix to some random face that is not parallel to or perpendicular to the c-axis (such as face b) and see that the light will be broken into two perpendicular rays, one vibrating parallel to the ω direction and the other vibrating perpendicular to an ε' direction. Thus we will see one of the random sections of the indicatrix.

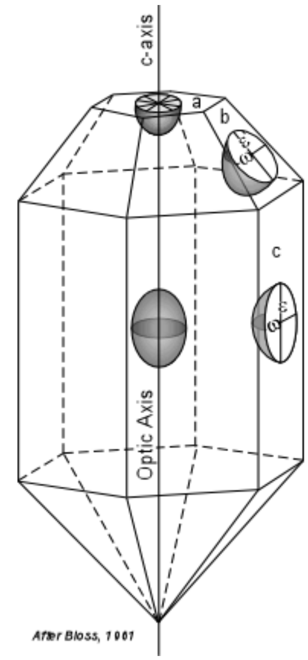

Figure 5. Indicatrix sections in different orientations of a hypothetical tetragonal mineral.

Biaxial Indicatrix

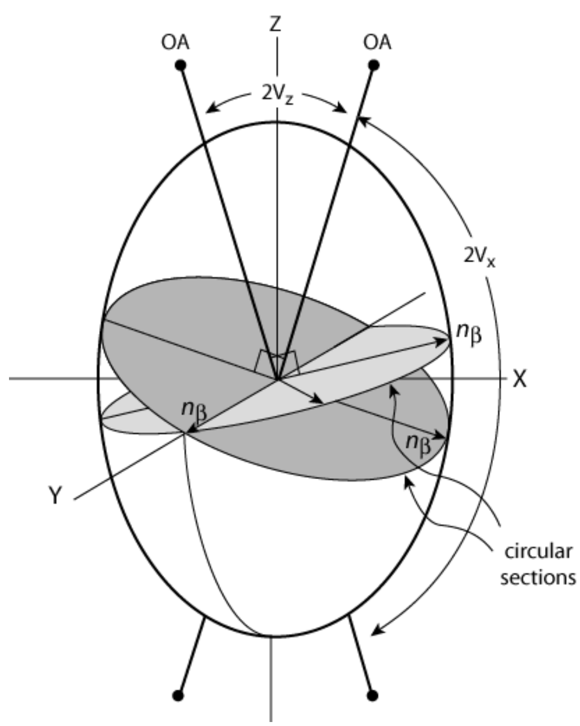

Figure 6. The Biaxial indicatrix, showing the principal indices of refraction and the location of circular sections and optic axes.

The **biaxial indicatrix**, like the isotropic and uniaxial indicatrices, diagrammatically illustrates the refractive index for vibration directions of light in biaxial minerals: in the *orthorhombic, monoclinic,* and *triclinic* crystal systems. The biaxial indicatrix has three principle axes, labeled α, β, and γ. Directions that have refractive indices between α and β, are referred to as α'. Directions with refractive indices between γ and β are referred to as γ' (**fig. 6**). Note that the β direction also must occur in the plane that includes α and γ. Similarly, if we were to draw all other possible planes that include the γ direction, β would have to occur in each of these as well. This results in two sections that would be circular with a radius equivalent to the β refractive index. These two sections are referred to as the circular sections. In **fig. 6** we see the two circular sections, each having a radius equal to the β refractive index.

Just like in uniaxial minerals, if one is looking down one of the **optic axes**, light traveling along the optic axis will be vibrating in the β direction, and thus the mineral would be extinct for all rotation positions.

Optic Sign of Biaxial Minerals

The optic sign of biaxial minerals depends on whether the β refractive index is closer to that of α or to γ. The **2V angle** is the angle between the optic axes. The **acute bisectrix** (**BX_A**) bisects the acute 2V angle. The **obtuse bisectrix** (**BX_O**) bisects the obtuse 2V angle.

Biaxial Negative

A mineral is **biaxial (-)** if the value of β is closer to γ than to α. In this case the acute 2V is bisected by the α refractive index direction. Thus we say that α is the acute bisectrix.

In the case of a biaxial negative mineral, 2Vα is the acute bisectrix, while 2Vγ is the obtuse bisectrix.

Biaxial Positive

A mineral is **biaxial (+)** if the value of β is closer to α than to γ. In this case the acute 2V is bisected by the γ refractive index direction. Thus we say that γ is the **acute bisectrix** (**BX_A**), because it bisects this angle. The obtuse angle between the optic axes is called the **obtuse bisectrix** (**BX_O**).

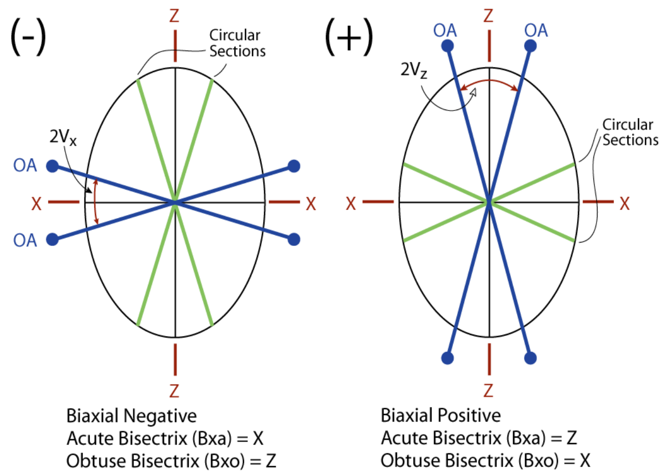

Figure 7. 2-D projection of the Biaxial indicatrix, with biaxial negative (-) on the left and biaxial positive (+) on the right. Note the difference in angles between the optic axes and the *z* axis.

Circular Section

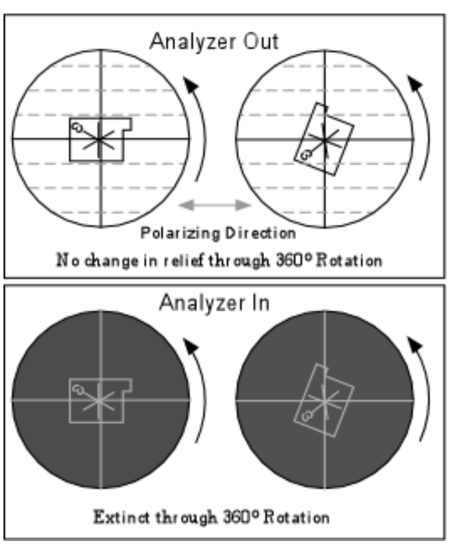

Figure 8. Microscope view of a uniaxial mineral aligned with the c axis parallel to the axis of the microscope, resulting in a circular section indicatrix.

If a crystal is mounted on the microscope stage with its optic axis oriented exactly perpendicular to the stage, the circular section of the indicatrix can be imagined to be on the upper surface of the crystal such as for the crystal face labeled a in **fig. 5.** In this orientation the crystal behaves just like an isotropic mineral.

Light polarized in an E-W direction entering the crystal from below remains polarized in an E-W direction as it passes through the crystal. Since light is vibrating parallel to an ω direction for all orientations of the grain, no change in relief would be observed as we rotate the microscope stage. With the analyzer inserted the grain would go extinct and would remain extinct throughout a 360° rotation of the microscope stage, because the light exiting the crystal will still be polarized in an E-W direction (**fig. 8**).

Principal Section

If the mineral grain is oriented such that the optic axis is oriented parallel to the microscope stage, then we can imagine the **principal section** of the indicatrix as being parallel to the top of the grain such as would be the case for a crystal lying on face c in **fig. 5**. In this case, the mineral will show birefringence for most orientations, unless one of the privileged directions in the crystal is lined up with the E-W polarizing direction of the incident light entering from below. *This when* ***extinction*** *occurs.*

As the refractive index will be will be different for the ω direction and the e direction, there will be some change in relief of the grain as it is rotated 90° between the two positions. How much change in relief would also depend on the birefringence ($\delta = n_s - n_f$) of the mineral (**fig. 9a**).

If the analyzer is inserted when the ω direction or the ε direction is parallel to the polarizing direction of the analyzer (N-S), the grain will be extinct, because the light will still be vibrating parallel to the polarizer as it emerges from the grain (**fig. 9b**).

If the ω and ε privileged directions in the crystal are at any other angle besides 0° and 90° to the polarizing direction, some light of the light will be vibrating at an angle to the polarizer on emergence from the crystal and some of this light will be transmitted through the analyzer (**fig. 9c**). This will be seen as color, called the **interference color**. Thus, as one rotates the stage with the analyzer inserted, the grain will go extinct every 90°, and will show an interference color between these extinction positions.

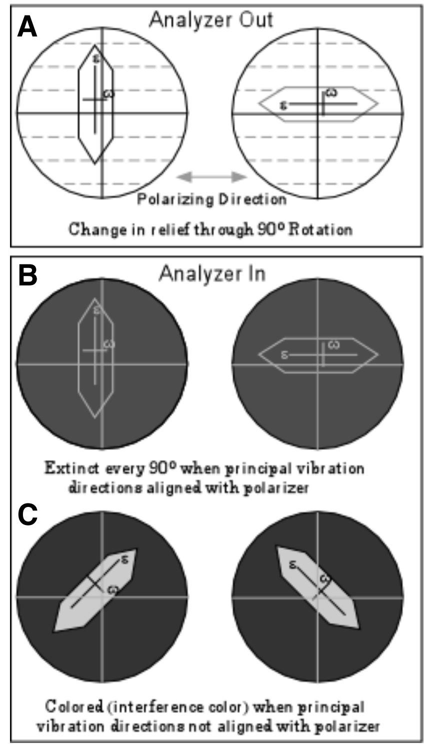

Figure 9. Microscope view of a uniaxial mineral aligned with the c axis parallel to the stage of the microscope, resulting in a principal section indicatrix.

Random Section

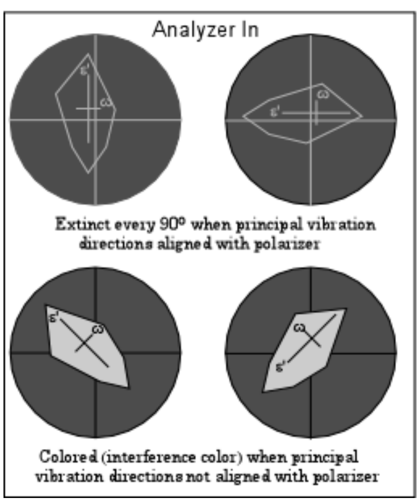

Figure 10. Microscope view of a uniaxial mineral with a random alignment of the optic axis, resulting in a random section indicatrix.

If the mineral grain is oriented such that the optic axis is oriented at an angle to the microscope stage, then we can imagine a random section of the indicatrix as being parallel to the top of the grain such as would be the case for a crystal lying on face b in **fig. 5**. In this case, the mineral will also show birefringence for most orientations, unless one of the privileged directions in the crystal is lined up with the E-W polarizing direction of the incident light entering from below.

But this time, one of the privileged directions corresponds to the ω direction and the other to an ε' direction in the crystal.

Thus, just as in the case of the principal section, as one rotates the stage with the analyzer inserted, the grain will go extinct every 90°, and will show an interference color between these extinction positions.